Accounting for Uncertainty

When the only thing certain is the date.

Sam Savage and Marc Van Allen

Today's crisis of confidence in the accounting industry has raised issues about the need for reform. In at least one area we believe the timing and the technology are right for significantly different accounting rules—the treatment of uncertainty in financial statements.

We start with some basic misconceptions about uncertainty, and show that these misconceptions are actually sanctioned by current accounting practice. There are analytical tools that can improve our perception of uncertainty, and there are some legal consequences of relying on currently sanctioned but flawed accounting standards.

UNDERSTANDING UNCERTAINTY

Consider the nature of uncertainty, and how financial statement users can and could think about it. In science, a number reflecting the outcome of future uncertain events is represented, not by a single estimate, but by a shape known as a *distribution*.

This may be thought of as a bar graph in which the horizontal axis represents the possible values the number can take on, and the vertical axis represents the likelihood that the number will actually take on that particular value. The net present value of an uncertain venture, for example, might have the shape as in Exhibit 1, which indicates various modes of success and failure as well as ranges of outcomes within each mode.

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EXHIBIT 1
NPV of Uncertain Venture

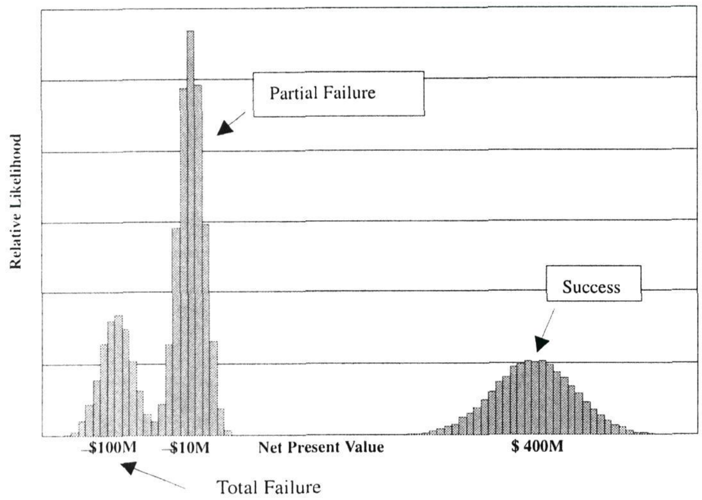
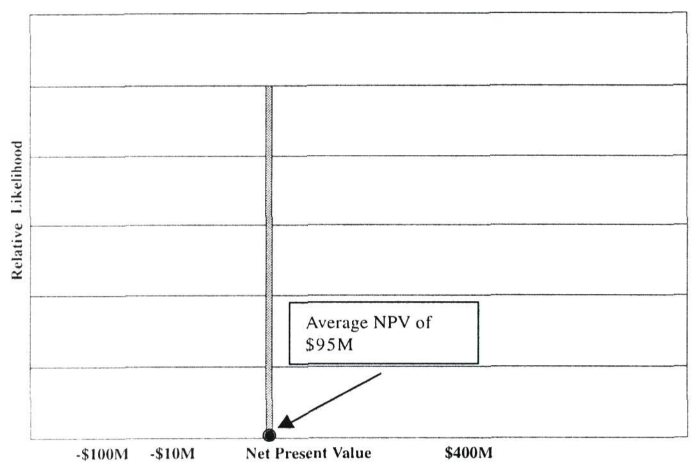


EXHIBIT 2
Average NPV of Uncertain Venture



THE FLAW OF AVERAGES

The view of an uncertain number as a shape is fundamental to virtually all successful modern technologies including the theory of finance. Yet in corporate financial statements, companies typically use single-point estimates of uncertain numbers to represent the true shape of things to come.

This is the case even though these point estimates require management to make assumptions about matters that are highly uncertain. These assumptions—largely absent from the pages of the financial statement—are legion. In fact, on a company's balance sheet, "the line about cash is the *only* line that does not reflect assumptions" of management ("Heard on the Street," *Wall Street Journal*, July 3, 2002). As a result, a company's point estimates (and assumptions hidden behind them) largely determine the contents of the financial statement.[1]

In replacing a shape with a single number, it is common to use the *average*, also known as the *expected value*, which for the NPV example in Exhibit 1 is $95 million, as shown in Exhibit 2.[2]

Replacing a distribution with a single bar, representing the average (or any other single number, for that matter) is fundamentally flawed. This problem in general has been dubbed the *flaw of averages*, and it comes in many forms (see Savage [2002]). We describe two broad categories. The first involves portfolio effects and the second non-linearity.

Portfolio Effects

One form of the flaw of averages results from the fact that the average of an uncertain number gives no indication of its degree of uncertainty or risk. The Nobel Prize-winning portfolio theory of Harry Markowitz [1959] and William Sharpe [1970] implies that when two assets have the same average value, the market will place greater value on the one with less risk. Thus, representing the uncertain situation in Exhibit 1 by the certain $95 million in cash represented in Exhibit 2 implies greater and unwarranted value for the entity.

Portfolio effects involve two separate phenomena: diversification and statistical dependence. By improperly accounting for these phenomena, generally accepted accounting principles create numerous inconsistencies. Estimated liabilities or *allowances* provide a great example.[3]

Almost all companies estimate the effects of adverse developments in their financial statements with reductions in carrying value for bad inventory and receivables or with estimated liabilities for anticipated restructuring charges, or warranty claims. According to a Baruch College professor of accounting, companies have "huge discretion in how big or small the reserves [allowances] are, establishing their size based not on hard, cold fact, but on educated guesses that pass for estimates" ("Heard on the Street," *Wall Street Journal*, July 3, 2002).

For our purposes, we focus on two types of estimates

EXHIBIT 3
Case 1—Single Customer

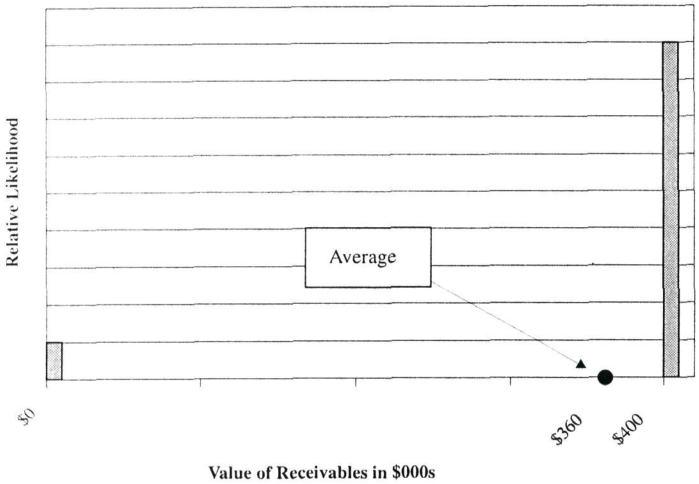
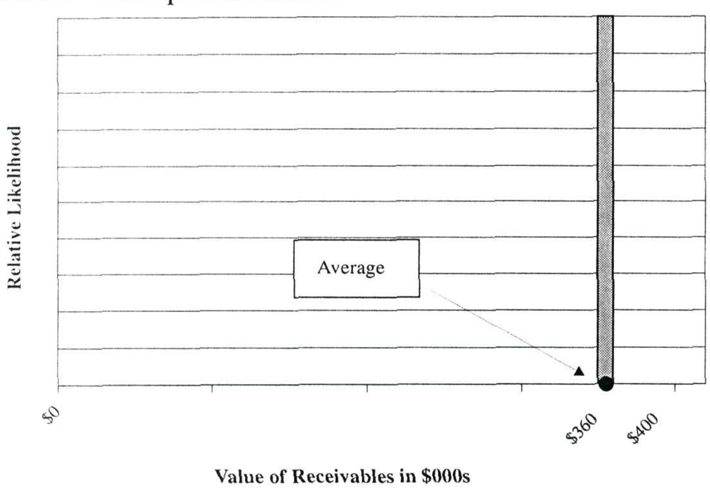


EXHIBIT 4
Case 2—Multiple Customers



of adversity: 1) for bad accounts receivable; and 2) for contingent legal claims.

Bad Accounts Receivable. A company is typically required to estimate the net realizable value of its accounts receivable. According to GAAP, accounts receivable are presented in the balance sheet as gross receivables less an allowance for bad debt (Siegel, Levine, and Qureshi [2001]). The allowance for bad debts is "an estimate of the amount of uncollectible accounts" (Stickney and Weil [1999]).

An example involving the valuation of accounts receivable is adapted from Johnson et al. [1993]. It addresses rules for recording uncertain numbers in financial statements. The aforementioned article provides a litany of violations of the flaw of averages sanctioned by the Financial Accounting Standards Board (FASB).

Case 1—Single Customer: A firm has $400,000 in receivables with a single customer who has a 90% chance of paying in full and a 10% chance of defaulting.

Case 2—Multiple Customers: A firm has $100 in receivables with each of 4,000 customers, all of whom have an independent 90% chance of paying in full and a 10% chance of defaulting.

We compare these two situations through their distributions as depicted in Exhibits 3 and 4. In case 1, the firm receives either the full $400,000 or nothing. The average, or expected value, of $360,000 is depicted by the solid circle. In case 2, there could be numerous outcomes, depending on exactly how many of the 4,000 customers default. Due to the law of large numbers, however, virtually all possible outcomes would lie within a $5,000 range on either side of $360,000. Given the resolution of the Exhibits, this places all outcomes in a single bar on the graph.

So here are two assets with the same average value. What are their market values?

According to modern portfolio theory, the diversified receivables of case 2 would have more market value than the single large receivable in case 1.

What are their values under GAAP? For case 2, GAAP recognizes that it will not be possible to collect the entire $400,000 of receivables, and the assets are correctly given an expected value of $360,000. For case 1, because there is only a 10% chance of default, GAAP states that "it is not probable that an asset has been impaired" (Johnson et al. [1993]). Therefore, on the books, the asset with the lower market value is worth... the full $400,000.

GAAP is supposed to reduce the likelihood of mischief by management, but when the GAAP book value and the real market value of two types of assets are out of kilter, it can have the opposite effect. Suppose a firm's only asset consists of the receivables of type 2, which have both a book value and a market value of $360,000. Now imagine managers heavily compensated by stock options, who have an incentive to increase the firm's paper value even if this reduces its real value.

EXHIBIT 5

Contingent Liability on the Books as Zero

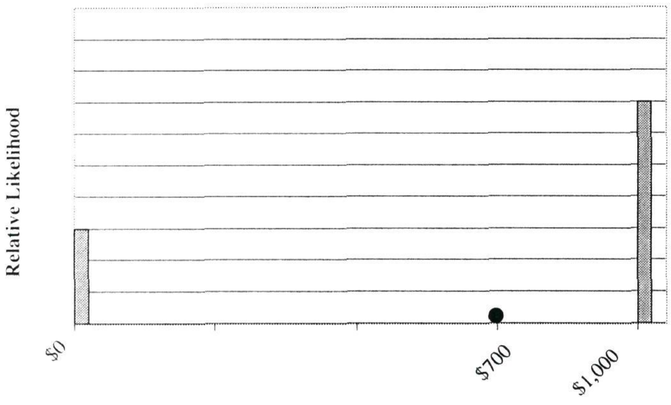
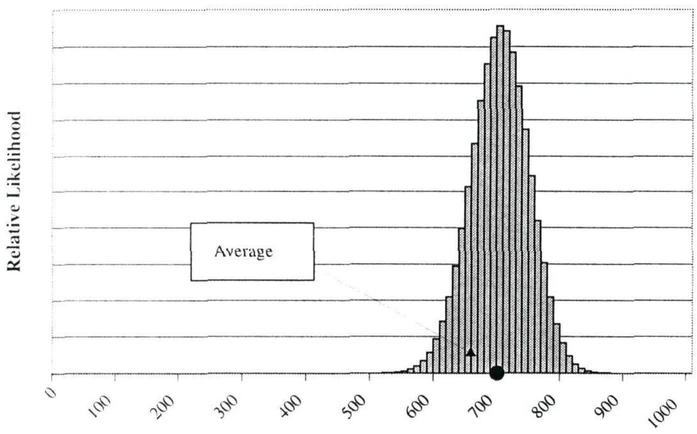


EXHIBIT 6

Certain Loss on the Books as Zero



The firm swaps its type 2 asset for another firm's type 1 asset. This should be easy, given type 2's higher market value. The firm's paper value has increased while its real value has declined. This makes it easier for it to raise more funds to pursue more such bad deals that drive the paper value even higher and their real value even lower, without ever violating GAAP.[4]

Contingent Legal Claims. FASB Statement No. 5, Accounting for Contingencies, provides the general framework for the recognition, measurement, and disclosure of contingent liabilities from legal claims. Statement No. 5 requires that "the estimated loss from a contingency be accrued by a charge to income if it is probable that a liability has been incurred and the amount of the loss can be reasonably estimated."

For contingent legal claims, the company must typically obtain an opinion letter from its attorneys regarding "the degree of probability of an unfavorable outcome" (SAS No. 12). The American Bar Association (ABA) instructs lawyers to opine that an unfavorable outcome is probable only if "the prospects of the [company] not succeeding are judged to be extremely doubtful and the prospects for success by the [company] in its defense are judged to be slight" ("Statement of Policy Regarding Lawyers' Responses to Auditors' Requests for Information" [1975]).

Assume a company faces litigation with two potential outcomes: a win (it suffers no liability); or a loss (it suffers a $ 1 billion liability). An attorney may determine that the company's chances of winning are better than "extremely doubtful" or "slight" (say there is a 30% chance of a winning).

In that event, as we interpret FASB Statement No. 5, the lawyer will not opine that losing (a 70% probability) is probable. The company's financial statements would value this liability at zero, and there would be no charge to earnings, even though the expected loss is $700 million. This is analogous to case 1 of the accounts receivable example, but even more far-fetched in that GAAP evaluates the situation according to the low-probability event (Exhibit 5).

As with receivables, a company's legal claims generally constitute a portfolio of liability risks. Consider, for example, a firm that is currently litigating 100 suits, each with potential damages of $10 million, and each with an independent probability of loss of 70%. FASB Statement No. 5 does not heed the law of large numbers, but operates at the atomistic level of individual risk. Instead of measuring the aggregate portfolio of risk, a company would repeatedly apply the *probable* test to each individual legal claim, and then sum up the results.

For the litigation case, this firm would apparently be in compliance with Statement No. 5 if it reports no liability, even though it faces a virtually certain loss of between $550 and $850 million, depending on exactly how many of the 100 cases it wins or loses (Exhibit 6).

GAAP Report Card. The two cases involving contingent legal claims are mathematically similar to the two cases involving accounts receivable. We compare all four below. Note that instead of making the same mistake twice, GAAP allows for a variety of mistakes to be built upon the same misrepresentation of uncertainty.

Case	*GAAP Valuation*
Single Account Receivable	Overestimates expected value of receivables by ignoring a low-probability event.
Multiple Accounts Receivable	Yields correct expected value.
Single Legal Claim	Erroneously indicates zero expected liability by ignoring a high-probability event.
Multiple Legal Claims	Erroneously indicates zero expected liability in the light of certain loss.

We have assumed statistical independence in the various uncertainties in these examples. If there are dependencies, the risks could be quite different—and the concept of statistical dependence is not addressed at all by GAAP in this context.

Non-Linearity

Non-linearity implies that the value of a calculation based on average assumptions is *not* the average value of the calculation. This is technically known as Jensen's inequality.

For example, the value of an out-of-the-money call option based on average stock price is zero, but zero is *not* the market value of the call option when averaged over all potential market moves. Pursuit of this topic led to the well-known Black/Scholes [1973] option pricing formula, and eventually to another Nobel Prize in economics.

In a more sobering example, suppose a drunk is staggering back and forth in traffic on a busy highway. His average position is the center line, and his state at his average position is alive. But the average state of the drunk is still dead.

Depreciation over an Uncertain Lifetime. Another example in Johnson et al. [1993] involves the use of averages in the denominator of a fraction.[5] We present our own numerical version of this problem here. Suppose a firm has a piece of equipment that is equally likely to last from one to ten years. Then on average it will last $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)/10 = 5.5$ years. What depreciation rate should be used?

The formula for the depreciation rate for an asset lasting Y years is (1/Y) per year. So the average depreciation would be $1/(5.5) = 18.2\%$, right? Wrong.

Exhibit 7 shows the depreciation rates of each of the ten possible outcomes, along with the average depreciation rate, which turns out to be 29.3%. Thus we see that the depreciation of the average *lifetime* is not the average depreciation.

Exhibit 8 displays the non-linear relationship between lifetime and depreciation rate.

Avoiding Insolvency. One of the harshest forms of non-linearity in financial statements involves projecting a company's ability to service debt without defaulting. Consider a company that expects receipts of $500,000 per

EXHIBIT 7
Depreciation Rates

Lifetime in Years (Y)	Depreciation Rate (1/Y)
1	100%
2	50%
3	33%
4	25%
5	20%
6	17%
7	14%
8	13%
9	11%
10	10%
Average 5.5	29.3%

EXHIBIT 8
Relationship of Lifetime and Depreciation Rate

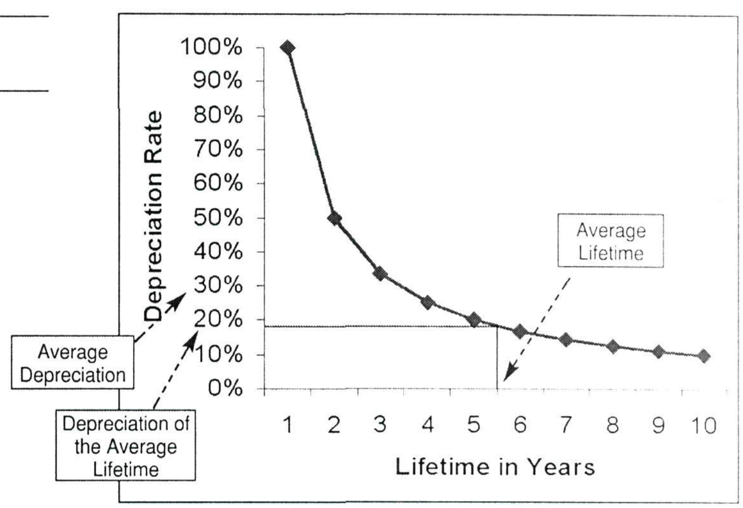
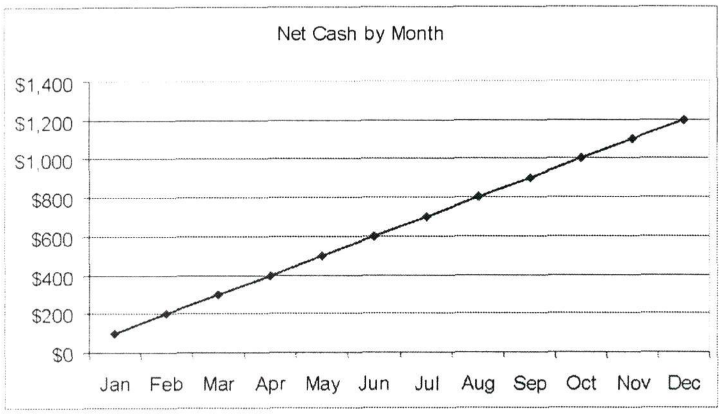


EXHIBIT 9
Net Cash by Month



month and expenditures of $400,000 including debt payments. Assuming that the company exactly meets each month's target for receipts and expenditures, starting from a zero cash position, its net cash would grow linearly as shown in Exhibit 9. This of course ignores any possible fluctuations in receipts or expenditures.

What if there is uncertainty in both receipts and expenditures? Suppose the firm knows that the average receipts and expenditures will be $500,000 and $400,000 per month, respectively, but that there will be month-to-month fluctuations in these numbers as reflected in the distributions in Exhibits 10 and 11? It is just this sort of uncertainty that requires funds from equity investors to avoid the potential of insolvency.

But how large an investment is required? There is no way to assess this using single-number estimates, but Monte Carlo simulation can provide a rational basis for answering this question.

Monte Carlo Simulation. Instead of using a single best guess as the input assumption, Monte Carlo simulation keeps uncertainty alive by bombarding a model with thousands of random inputs while tracking the resulting outputs. Developed during the Manhattan project, the technique is used extensively to model uncertain behavior in fields as diverse as physics, engineering, health care, and finance. It was introduced to the electronic spreadsheet in the mid 1980s, and continues to become more compelling with increasing computer power.

In the receipts-expenditures-investment example, Monte Carlo simulation can quickly provide insight into the risk of default and the optimal debt-to-equity ratio. First, we create a 12-month spreadsheet model of cash flow, where the cash position at the end of any month equals the cash position from the previous month plus the receipts from the current month minus the expenditures from the current month.

Next, receipts and expenditures for each month are drawn randomly, according to the distributions in Exhibits 10 and 11. Each random draw results in a different trajectory of cash flows. Four of these are displayed in Exhibit 12. Note that in one trajectory the enterprise becomes insolvent in March.

We repeat the last step for thousands of iterations while tallying the percentage of times that the enterprise becomes insolvent. For the distributions in Exhibits 10 and 11, for example, the probability of insolvency within the first 12 months is about 30%.

Here we have another classic case of the flaw of averages. The assumption that the firm will always hit its receipt and expenditure targets is analogous to the assump-

EXHIBIT 10
Distribution of Monthly Receipts

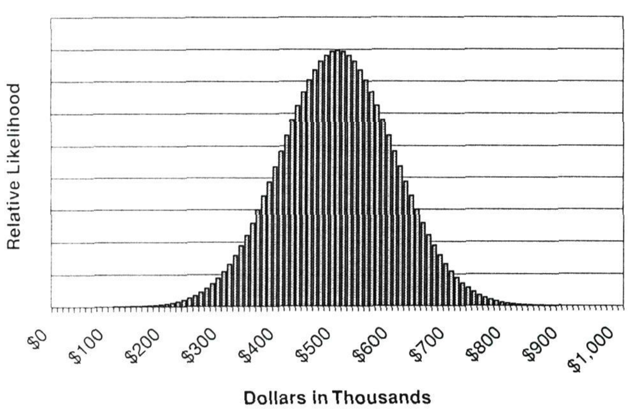


EXHIBIT 11
Distribution of Monthly Expenditures

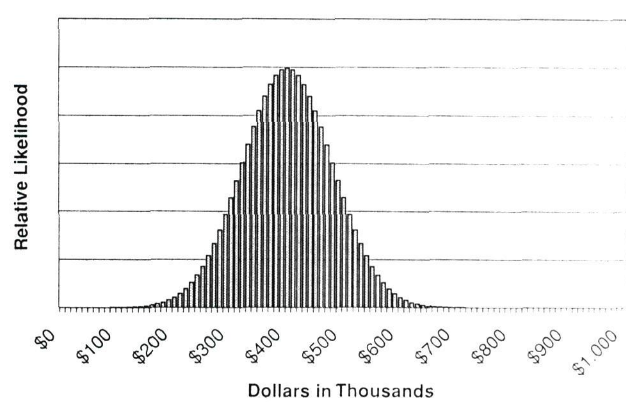
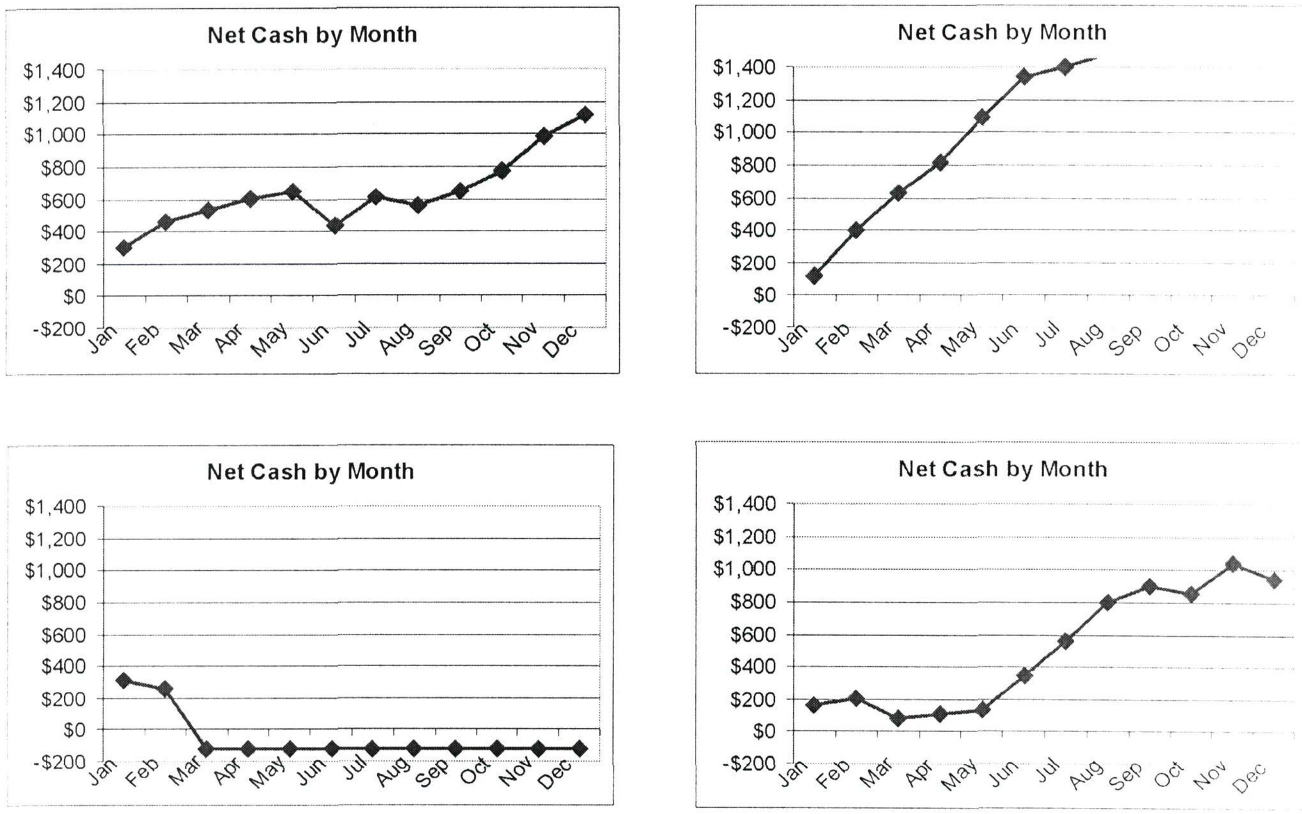


EXHIBIT 12
Monte Carlo Cash Flow Estimates



tion that the drunk will always be on the center line of the highway. Or, to put it another way, the chance of insolvency, given average receipts and expenditures each month, is zero, but the chance of insolvency averaged over all possible receipts and expenditures is 30%.

This experiment may be repeated for various levels of shareholder equity investment to see how the likelihood of insolvency is impacted. Exhibit 13 shows that a January cash infusion from equity investors of $100,000 reduces the risk of insolvency by two-thirds, and that an infusion of $300,000 reduces risk to under 2%, but a shareholder investment in excess of $300,000 provides almost no additional risk reduction.

Shaking the Ladder. It has been said that a Monte Carlo simulation is only as good as the probability distributions upon which it is based. We would disagree with the conclusion that simulation is not valuable unless the distributions are accurate.

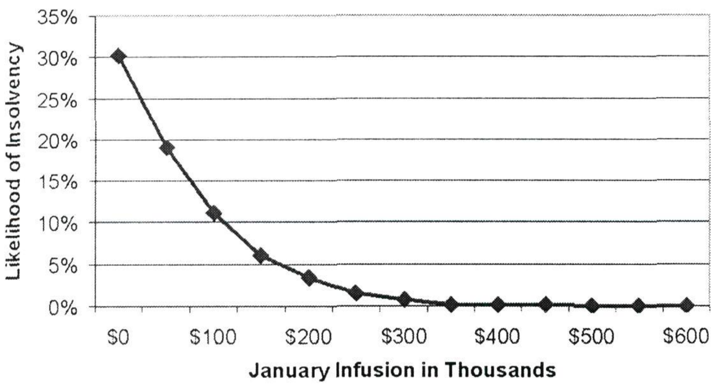
Consider as an analogy shaking a ladder to check its stability before you climb on it. This constitutes a Monte Carlo simulation that subjects the ladder to random forces. Although the distribution of forces on a ladder when you shake it is not the same as the distribution of forces when you climb on it, this should not discourage you from continuing to shake ladders.

Similarly, virtually any reasonable distribution of receipts and expenditures will lead to some likelihood of insolvency without a cash infusion. Furthermore, the resulting graph of likelihood versus extent of infusion will look like Exhibit 13. Any set of point estimates where receipts exceed expenditures, however, will not even admit to the possibility of insolvency.

LEGAL REQUIREMENTS

The investing public relies on financial statements to accurately depict a company's financial condition. Under the securities laws, "fair presentation is the touchstone for determining the adequacy of disclosure in financial statements."[6]

EXHIBIT 13
Investment Needs



Yet throughout the last decade, this principle has gone largely unheeded. In fact, some companies have seemed to believe that they could discharge their obligations by "checking the boxes"—that is, by technically complying with GAAP, whether or not the accounting treatment used accurately reflects the economic reality of the transaction or situation.

The law requires more. This point was hammered home in a landmark case in 1969.[7] The defendants in this case claimed that criminal charges were foreclosed because the financial statements literally complied with GAAP. In rejecting this argument, the Second Circuit Court of Appeals held that if literal compliance with GAAP creates a misleading impression in the minds of shareholders, the defendants could and would be held liable.

ROAD TO REFORM

When issues of risk and uncertainty are involved, GAAP accounting gets tripped up by the flaw of averages. In these cases, literal compliance with GAAP may *not* fairly present a company's financial condition as the law requires. An explicit modeling of uncertainty using Monte Carlo simulation or related techniques would help a company avoid many of the pitfalls of the flaw of averages.

Some in the accounting industry are already familiar with Monte Carlo. How might it gain more widespread adoption? Even the concept of net present value was embraced slowly.

We do see a possible evolutionary path to reform. As of this writing, we are aware of two widely adopted Microsoft Excel-based Monte Carlo packages.[8] These programs have been used for over a decade in numerous firms to model various types of corporate risks. More recently, investors themselves have adopted Monte Carlo to illuminate the potential behavior of their investments. Firms working in this area include American Express, Bessemer Trust, and Financial Engines (cofounded by Nobel Laureate Sharpe).

Thus Monte Carlo is already being used to model corporate risk from within and without. It would be a relatively small step to use Monte Carlo models as a medium of communication between firms and investors. A potential starting place is in the disclosures of specific risks that accompany financial statements. Simple spreadsheet-based simulation models would better express various forms of risk than paragraphs of technical explanation.

In the final analysis, the marketplace will decide whether Monte Carlo or any other analysis can improve investor confidence in our accounting systems. But without question, Monte Carlo reveals information that remains hidden in the bias of point estimates. And in the long run, the marketplace tends to reward the release of information.

ENDNOTES

The authors gratefully acknowledge the assistance of Roman Weil and helpful comments from June Klein, Allan Kretz, and William Wecker.

[1]According to Roman Weil (in personal correspondence): "Thoughtful accountants, going as far back as Robert Elliott, intellectual leader of KPMG for many years, point out that even cash is uncertain, once you take into account amounts denominated in foreign currencies and the need to estimate exchange rates on the balance sheet date. Elliott claims the only number on the balance sheet about which there is no uncertainty is ... [think about it] ... [think some more] ... the date itself" (brackets in the original).

[2]If the bars in Exhibit 1 were made of a dense solid material and placed on a very light beam, this is where the shape would balance.

[3]Common business language refers to these as *reserves*, but laypeople so misunderstand this word (thinking of it as a pool of funds, something with a debit balance, rather than as something with a credit balance) that we avoid using the word at all.

[4]This is an argument for compensating management using Asian options that reward increases in average stock price over an extended period instead of price spikes.

[5]Additional examples of this form of the flaw of averages include: 1) that the cost of the pension for an employee's expected life is *not* the expected cost of the pension for the

employee's life; and 2) that the percentage of a job completed assuming the expected labor hours available is ***not*** the expected percentage of the job that will be completed. Johnson et al. [1993] point out that actuaries who do these calculations typically get Jensen's inequality right; it's lay thinkers about the issue who get it wrong.

[6]*Herzfeld v. Laventhol,* 378 F. Supp. 112, 122 (S.D.N.Y. 1974), affirmed 540 F.2d 27 (2d Cir. 1976).

[7]*United States v. Simon,* 425 F.2d 796 (2d Cir. 1969), certiorari denied, 397 U.S. 1006 (1970).

[8]The program @RISK is from Palisade Corp. (www.Palisade.com). Crystal Ball® is from Decisioneering Inc. (www.Decisioneering.com). A simpler Monte Carlo package designed by Dr. Savage for teaching purposes and smaller applications is XLSim® from AnalyCorp Inc. (www.AnalyCorp.com).

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